Advanced Ionospheric Modeling

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Outline

• Origin and phenomenology of the ionosphere
• Effect of the ionosphere on GPS signals
• Effect of the ionosphere on network solutions
• Formulation of the ionospheric model
• Model results
  – State of the ionosphere
  – Ambiguity resolution
• Summary
The origin of the ionosphere

schematic, not to scale

Ionization of molecules

Gravity

Ionosphere

Troposphere
Electron density as function of altitude

A typical output of the International Reference Ionosphere (combination of measurement and sophisticated modeling)

http://nssdc.gsfc.nasa.gov/space/model/ionos/iri.html

Strongly peaked at a particular altitude

Integrated value: Vertical TEC (VTEC)
The origin of the ionosphere

- Gravity keeps gas molecules in the vicinity of earth's surface, forming the atmosphere.
- Radiation from the sun ionizes a small fraction of the gases, forming a dispersive medium, the ionosphere.
- Ionosphere experiences disturbances depending on:
  - solar activity (11 year solar cycle)
  - geomagnetic disturbances, solar winds
  - geographic location of the sun relative to earth
  - air currents (winds) in the upper layers of the atmosphere
11 year solar cycle, observed number of sunspots

http://science.nasa.gov/ssl/pad/solar/sunspots.htm
11 year solar cycle, predicted number of sunspots

http://science.nasa.gov/ssl/pad/solar/sunspots.htm

rough times are coming up soon again

http://sidc.oma.be
A global VTEC map, **calm ionosphere**

ESA Ionospheric Monitoring Facility
http://nng.esoc.esa.de/gps/ionmon.html

Vertical Total Electron Content on a global map, as derived from GPS base station observations
A global VTEC map, active ionosphere

Center for Orbit Determination in Europe (CODE)
http://www.aiub.unibe.ch/ionosphere
Complex dynamical behavior also on small scales

DLR Institute for Communications and Navigation

Vertical Total Electron Content over Europe Oct. 7 2005, as derived from GPS base station observations.
Effects of charged gas on radio signals

schematic, not to scale
Effects of ionosphere on GPS signals

• Between satellite and rover
  – the number of phase cycles decreases (phase velocity increases)
  – the number of code chips increases (code velocity decreases)
  – by the same amount if expressed in units of length

• Magnitude of the effect is
  – proportional to the density of electrons integrated over the path of signal propagation (total electron content - TEC)
  – proportional to the square of the carrier wavelength (dispersive medium): Great tool for multi-frequency signals
A case study: BLVA-network

49.2 km

74.9 km
Electron density, ionospheric layer, pierce points and mapping function

\[ TEC = VTEC \times m = VTEC \times \frac{1}{\cos \varphi} \]

Enhancement of 'seen' electron density along the propagation path: mapping function \( m \)

IRI - 2001

Reference Stations
Modeling gradients of the ionosphere

Taylor expansion of Ionospheric effect across the network projection to the ionosphere

\[ I(\lambda, \phi) = I_0 + a_\lambda \Delta \lambda + a_\phi \Delta \phi + \ldots \]

Gradients:

\[ a_\lambda = \frac{\partial I}{\partial \lambda} \quad a_\phi = \frac{\partial I}{\partial \phi} \]
Modeling gradients of the ionosphere

Taylor expansion of Ionospheric effect across the network projection to the ionosphere

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Gradients

\[ a_\lambda = \frac{\partial I}{\partial \lambda} \quad a_\phi = \frac{\partial I}{\partial \phi} \]
Piercepoint geometry and dynamical evolution

pierce point coordinates in geocentric coordinates during a period of 6 hours

Latitude (degrees)  Longitude (degrees)

20 minutes between frames
Piercepoint geometry
and dynamcial evolution
pierce point coordinates in solar centered coordinates
throughout the course of a day

TEC values obtained from CODE
http://www.aiub.unibe.ch/ionsphere
distance traveled by signal
\[ r = \lambda_q (\phi_q + N_q) \]
where \( \lambda_q \) is the wavelength, \( \phi_q \) is the phase, and \( N_q \) is the integral ambiguity.

Using
\[ I_q \propto \frac{1}{f_q^2} \]
and forming difference for \( q = 1, 2 \)

\[ \frac{\lambda_1^2}{\lambda_2^2 - \lambda_1^2} (\lambda_1 \phi_1 - \lambda_2 \phi_2) = -\frac{\lambda_1^2}{\lambda_2^2 - \lambda_1^2} (\lambda_1 N_1 - \lambda_2 N_2 - MP_1 + MP_2) + I_1 + \epsilon \]

Measurement \( \phi^* \)

State variables \( (N^*, MP^*, I_1) \)
(Ambiguity, Multipath, Ionosphere)

Model error + system noise
Kalman filter – part 1: Observation equation

\[ \vec{l} = A \vec{x} \]

Observation, measurement \[ \vec{l} = (\phi_1^*, \ldots, \phi_N^*) \]

measurements from N stations

State vector \[ \vec{x}_i = (N_1^*, \ldots, N_N^*, MP_1^*, \ldots, MP_N^*, I, a_\lambda, a_\phi)^T \]

N ambiguities N stations
N multipaths N stations
3 parameters 1 ionosphere

Design matrix
\[
A = \begin{pmatrix}
-1 & 0 & 1 & 0 & \cdots & \cdots & \cdots & m_1 & m_1 \Delta \lambda_1 & m_1 \Delta \phi_1 \\
0 & \cdots & 0 & 1 & \cdots & \cdots & \cdots & m_N & m_N \Delta \lambda_N & m_N \Delta \phi_N
\end{pmatrix}
\]

mapping function relative distance to reference piercepoint
Kalman filter – part 2:
Time update

\[ \tilde{x}_{i+1} = \Phi \tilde{x}_i + \tilde{w}_i \]

Estimated state for next epoch \( \tilde{x}_{i+1} \)
State of current epoch \( \tilde{x}_i \)

State vector
\[
\tilde{x}_i = (N_1^*, ..., N_N^*, MP_1^*, ..., MP_N^*, I, a_\lambda, a_\phi)^T
\]

Time update matrix
\[
\Phi = \begin{pmatrix}
I_{N \times N} & 0 & 0 & 0 & 0 & 0 \\
0 & I_{N \times N} e^{-t/t_c} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & \Delta \lambda_{CPP} & \Delta \phi_{CPP} \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
\end{pmatrix}
\]

System noise
\[
E [\tilde{w}^T \tilde{w}] = R
\]

Motion of reference point
Parameters of the calculation
Same set of parameters applied for all networks around the globe
Components of the state vector

\[ \mathbf{x}_i = (N_1^*, \ldots, N_N^*, MP_1^*, \ldots, MP_N^*, I, a_\lambda, a_\phi)^T \]

- **Ambiguity (meters):** Differences between ambiguities are constant.
- **Multipath (centimeters):** Correlated between days.
- **Ionosphere parameters:** Activity of ionosphere about to decline.

Use double differences together with code measurement and tropo model to resolve the ambiguity (FAMCAR).
Ionospheric state parameters from the bavarian sub network

CODE TEC Map (schematic)

Observation data: BLVA Day 068, 2002
Ionospheric state parameters from a Japanese network

Observation data: GSI Day 019, 2002

CODE TEC Map (schematic)
Contributions of first and second order correction terms

Typical orders of magnitude for first and second order derivatives

**linear terms**

\[ O(a_\lambda) \sim O(a_\phi) \sim 1 \times 10^{-6} = 1 \text{ mm / km} \]

**second order**

\[ O(a_{\lambda\lambda}) \sim O(a_{\lambda\phi}) \sim O(a_{\phi\phi}) \sim 1 \times 10^{-12} \text{ m}^{-1} \]

**Network with extensions**

\[ O(\Delta \lambda) \sim O(\Delta \phi) \sim 100 \text{ km} \]

The corrections of the individual terms to the Ionospheric value is thus

**linear terms**

\[ O(a_\lambda \Delta \lambda) \sim O(a_\phi \Delta \phi) \sim 10 \text{ cm} \]

**second order**

\[ O\left( \frac{1}{2} a_{\lambda\lambda} \Delta \lambda^2 \right) \sim O(a_{\lambda\phi} \Delta \lambda \Delta \phi) \sim O\left( \frac{1}{2} a_{\phi\phi} \Delta \phi^2 \right) \sim 1 \text{ cm} \]

Depending on your application, higher order terms need to be taken care of, or can be neglected.
Application of the model in neworking software, ambiguity resolution

Tracked satellites and fixed satellites

Number of unfixed satellites

Zeroth order

First order

local time
Resolving ambiguities with different orders of approximation

<table>
<thead>
<tr>
<th>Network</th>
<th>Homogeneous Iono</th>
<th>First Order Iono</th>
<th>Second Order</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Japanese network (GSI):</strong></td>
<td>96.68%</td>
<td>97.73%</td>
<td>97.58%</td>
</tr>
<tr>
<td>stations</td>
<td>7</td>
<td>70 km x 60 km</td>
<td></td>
</tr>
<tr>
<td><strong>Swiss network (Swisstopo):</strong></td>
<td>98.11%</td>
<td>98.52%</td>
<td>98.57%</td>
</tr>
<tr>
<td>stations</td>
<td>29</td>
<td>310 km x 210 km</td>
<td></td>
</tr>
<tr>
<td><strong>German network (ASCOS):</strong></td>
<td>98.03%</td>
<td>99.29%</td>
<td>99.29%</td>
</tr>
<tr>
<td>stations</td>
<td>28</td>
<td>250 km x 350 km</td>
<td></td>
</tr>
</tbody>
</table>

The diagram shows the unfixed percentage for each network, with smaller values indicating better resolution. The unfixed percentage is used to evaluate the effectiveness of each order of approximation.
Summary

Model ionospheric phase advance in terms of Taylor series to the desired order across local area network

Extract ionospheric parameters, multipath and doubly differenced ambiguities by means of a Kalman filter

Obtain increased fixing performance and reliability for small to intermediate network sizes together with a physical picture of the evolution of the ionosphere
Some trigonometry leads to

\[ m = \frac{1}{\sqrt{1 - \left(\frac{R_E}{R_E + h \cos \varphi}\right)^2}} \]

The mapping function is depicted in the schematic, not to scale.
Illustration of PP Dynamics

\[
\tan x = \frac{\sin \Lambda_S}{R_E/R_S - \cos \Lambda_S}
\]

\[
\sin(\Lambda_{PP} + x) = \frac{R_E}{R_E + h} \sin x
\]
http://sidc.oma.be

SUNSPOT NUMBER Ri

- Monthly
- Smoothed

TIME (years)